A228 Iteration

Q1.

(a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \dfrac{1}{4} - \dfrac{x^3}{4}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{1}{4} - \dfrac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

....

(3)

(Total for question = 6 marks)

Q2.

(a) Show that the equation $x^3 - 3x^2 + 3 = 0$ has a solution between $x = 2$ and $x = 3$

(2)

(b) Show that the equation $x^3 - 3x^2 + 3 = 0$ can be rearranged to give $x = \sqrt[3]{3x^2 - 3}$

(1)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{3x_n^2 - 3}$ to find the value of x_2
Give your answer correct to 3 decimal places.

....

(3)

(Total for question = 6 marks)

Q3.

At the start of year n, the quantity of a radioactive metal is P_n
At the start of the following year, the quantity of the same metal is given by

$$P_{n+1} = 0.87P_n$$

At the start of 2016 there were 30 grams of the metal.

What will be the quantity of the metal at the start of 2019?
Give your answer to the nearest gram.

.... grams

(Total for question = 3 marks)

Q4.

(a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

(2)

(b) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \quad \text{with } x_0 = 3.2,$$

find the values of x_1, x_2 and x_3

....

(3)

(c) Explain what the values of x_1, x_2 and x_3 represent.

....

....

(1)

(Total for question is 6 marks)

Q5.

(a) Show that the equation $2x^3 + 4x = 3$ has a solution between 0 and 1

(2)

(b) Show that $2x^3 + 4x = 3$ can be rearranged to give $x = \dfrac{3}{4} - \dfrac{x^3}{2}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{3}{4} - \dfrac{x_n^3}{2}$ times to find an estimate for the solution to $2x^3 + 4x = 3$

....

(3)

(Total for question = 6 marks)

Q6.

(a) Show that the equation $x^3 + 5x - 4 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 5x - 4 = 0$ can be arranged to give $x = \dfrac{4}{x^2 + 5}$

(2)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{4}{x_n^2 + 5}$ twice, to find an estimate for the solution of $x^3 + 5x - 4 = 0$

....

(3)

(Total for question = 7 marks)

Q7.

The number of slugs in a garden t days from now is p_t where

$p_0 = 100$
$p_{t+1} = 1.06p_t$

Work out the number of slugs in the garden 3 days from now.

....

(Total for question = 3 marks)

Q8.

The number of bees in a beehive at the start of year n is P_n.
The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

....

(Total for question is 3 marks)

Q9.

(a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arrange to give $x = \dfrac{5}{x^2 + 7}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \dfrac{5}{x_n^2 + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

....

(3)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$, comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

....

....

(2)

(Total for question = 9 marks)

Q10.

Using $x_{n+1} = -2 - \dfrac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$x_1 = $ ....

$x_2 = $ ....

$x_3 = $ ....

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

....

....

....

(2)

(Total for question = 5 marks)