A228 Iteration

① a) $f(0) = -1$
$f(1) = 4$
change of sign ∴ at least 1 root

b) proof

c) $x_0 = 0$
$x_1 = \frac{1}{4}$
$x_2 = \frac{63}{256}$

② a) $f(2) = -1$
$f(3) = 3$
change of sign ∴ at least 1 root

b) proof

c) $x_0 = 2$
$x_1 = 2080\ldots$
$x_2 = 2.153\ldots$
∴ 2.153

③ 20 grams

④ a) proof

b) $x_0 = 3.2$
$x_1 = 3.29296875$
$x_2 = 3.276659786$
$x_3 = 3.279420685$

c) estimation of the solution after iteration

⑤ a) $f(0) = -3$
$f(1) = 3$
change of sign ∴ at least 1 root

b) proof

c) $x_0 = 0$
$x_1 = 0.75$
$x_2 = 0.5390625$
$x_3 = 0.671677351$

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⑥ a) $f(0) = -4$
$f(1) = 2$
change of sign ∴ at least 1 root

b) proof

c) $x_0 = 0$
$x_1 = 0.8$
$x_2 = 0.709921985$

⑦ 119

⑧ 10169 (better), accepted 10170

⑨ a) $f(0) = -5$
$f(1) = 3$
change of sign ∴ at least 1 root

b) proof

c) $x_0 = 1$
$x_1 = 0.625$
$x_2 = 0.6765327696$
$x_3 = 0.670448300\,1$

d) $f(0.670448300\,1) = -0.0054947\ldots$
accurate as it is close to 0

⑩ a) $x_0 = -2.5$
$x_1 = -2.64$
$x_2 = -2.57392$
$x_3 = -2.603767255$

b) $x^3 + 2x^2 + 4 = 0$ can be rearranged to give the iterative formula $x_{n+1} = -2 - \frac{4}{x_n^2}$

and iteration is an estimate of the solution